Bayesian I-optimal designs for choice experiments with mixtures

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Outline

1. Choice modeling and choice experiments
2. Mixture experiments
3. Combining choice models and mixture models
4. Optimality criteria for choice experiments
5. Results
6. Conclusions and future work

Discrete choice experiments

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- Example: a customer responding whether they prefer to buy product A, B or C
- Models assume a latent utility function used to derive the probability of each respondent making each decision

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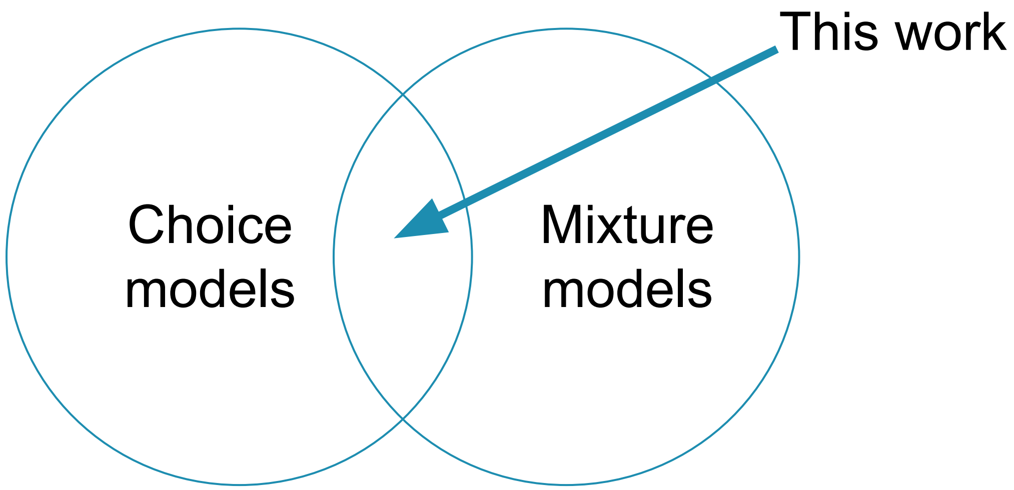
- For us, the characteristic of interest is the **preference** of respondents
- Choice experiments are ideal to collect data for quantifying and modeling preferences for mixtures

Choice experiments with mixtures



Choice experiments with mixtures

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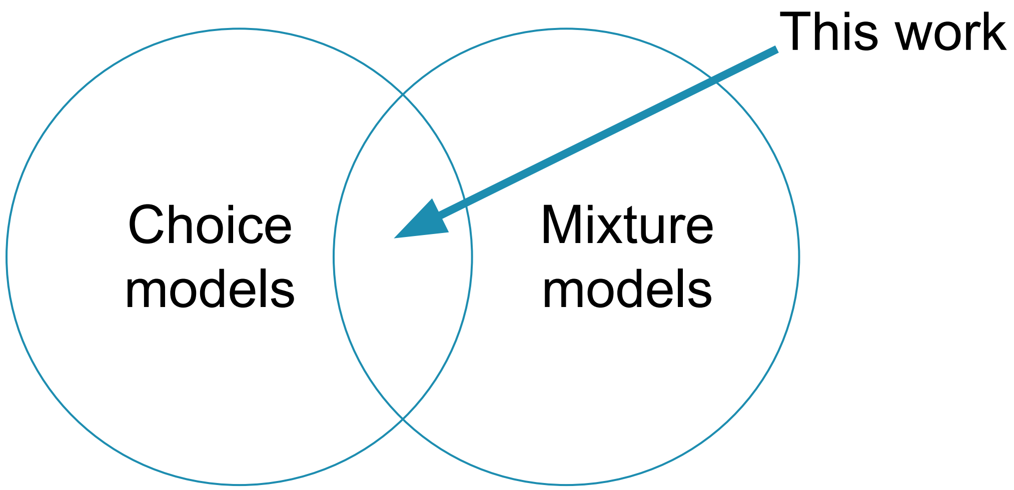

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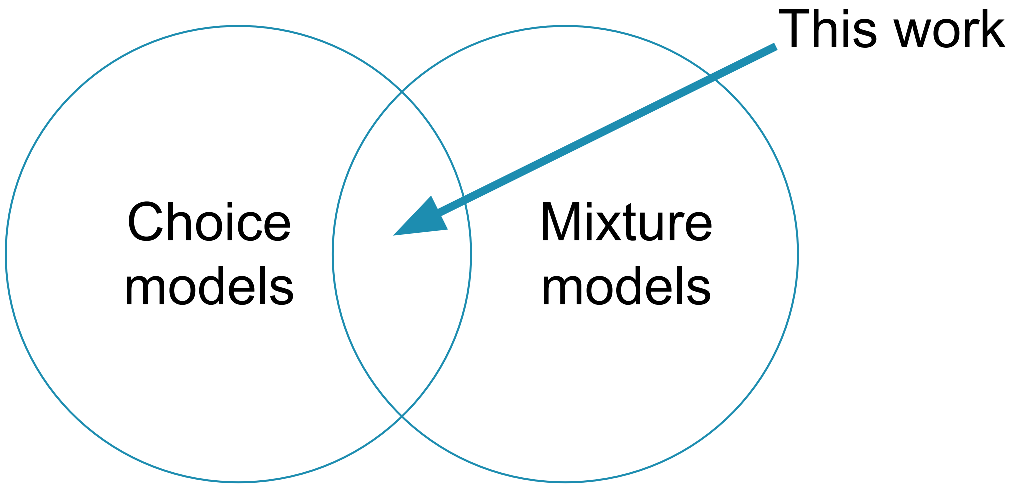

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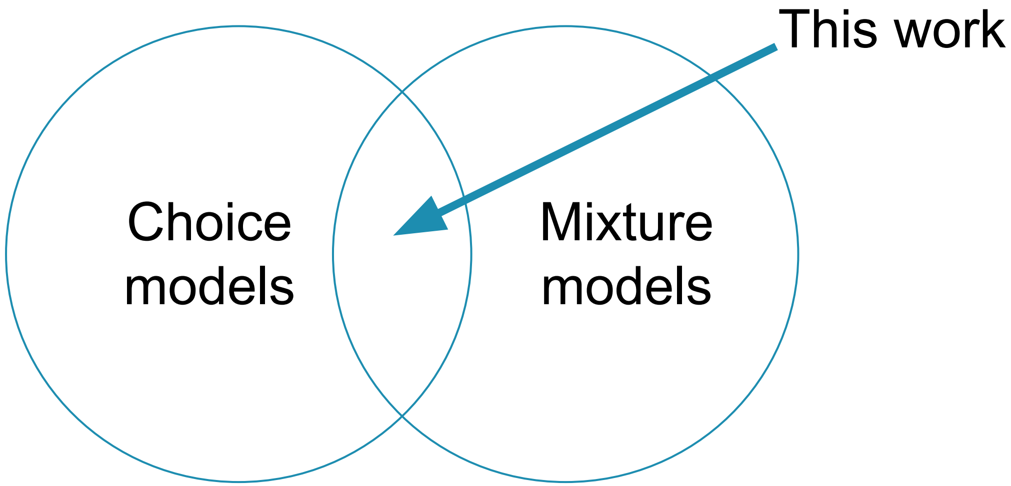
- Preferences for cocktails involving different proportions of mango juice, lime juice, and blackcurrant syrup
- Experimental data involved the responses of sixty people, each making eight pairwise comparisons of different cocktails



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- Optimal design of experiments is the branch of statistics that deals with the construction of efficient experimental designs

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- In experiments with mixtures, one wants to optimize the composition of the mixture to maximize consumer preference
- Precise predictions are crucial, hence I-optimal designs are more suitable

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- Dedicated models are needed to avoid perfect collinearity
- Special-cubic Scheffé model:

$$Y = \sum_{i=1}^{q} \beta_i x_i + \sum_{i=1}^{q-1} \sum_{j=i+1}^{q} \beta_{ij} x_i x_j + \sum_{i=1}^{q-2} \sum_{j=i+1}^{q-1} \sum_{k=j+1}^{q} \beta_{ijk} x_i x_j x_k + \varepsilon$$

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- The probability that a respondent chooses alternative $j \in \{1, ..., J\}$ in choice set s is

$$p_{js} = \frac{\exp\left[\boldsymbol{f}^T(\boldsymbol{x}_{js})\boldsymbol{\beta}\right]}{\sum_{t=1}^{J} \exp\left[\boldsymbol{f}^T(\boldsymbol{x}_{ts})\boldsymbol{\beta}\right]}$$

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- Perceived utility modeled as

$$U_{js} = \boldsymbol{f}^T(\boldsymbol{x}_{js})\boldsymbol{\beta} = \sum_{i=1}^{q-1} \beta_i^* x_{ijs} + \sum_{i=1}^{q-1} \sum_{k=i+1}^{q} \beta_{ik} x_{ijs} x_{kjs} + \sum_{i=1}^{q-2} \sum_{k=i+1}^{q-1} \sum_{l=k+1}^{q} \beta_{ikl} x_{ijs} x_{kjs} x_{ljs} + \varepsilon_{js}$$

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- Numerical approximation to Bayesian D-optimality criterion

$$\mathcal{D}_B \approx \log\left(\frac{1}{R}\sum_{i=1}^{R}\left[\det\left(\boldsymbol{I}^{-1}(\boldsymbol{X},\boldsymbol{\beta}^{(i)})\right)\right]^{\frac{1}{r}}\right)$$

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$$\mathcal{I} = \mathrm{tr}\left[\boldsymbol{I}^{-1}(\boldsymbol{X}, \boldsymbol{\beta})\boldsymbol{W}_u\right]$$

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- Ruseckaite et al. obtained a prior distribution for parameter vector $\boldsymbol{\beta}$ in a special-cubic Scheffé model
- We used the same prior distribution to compute Bayesian D- and I-optimal designs using a coordinate-exchange algorithm

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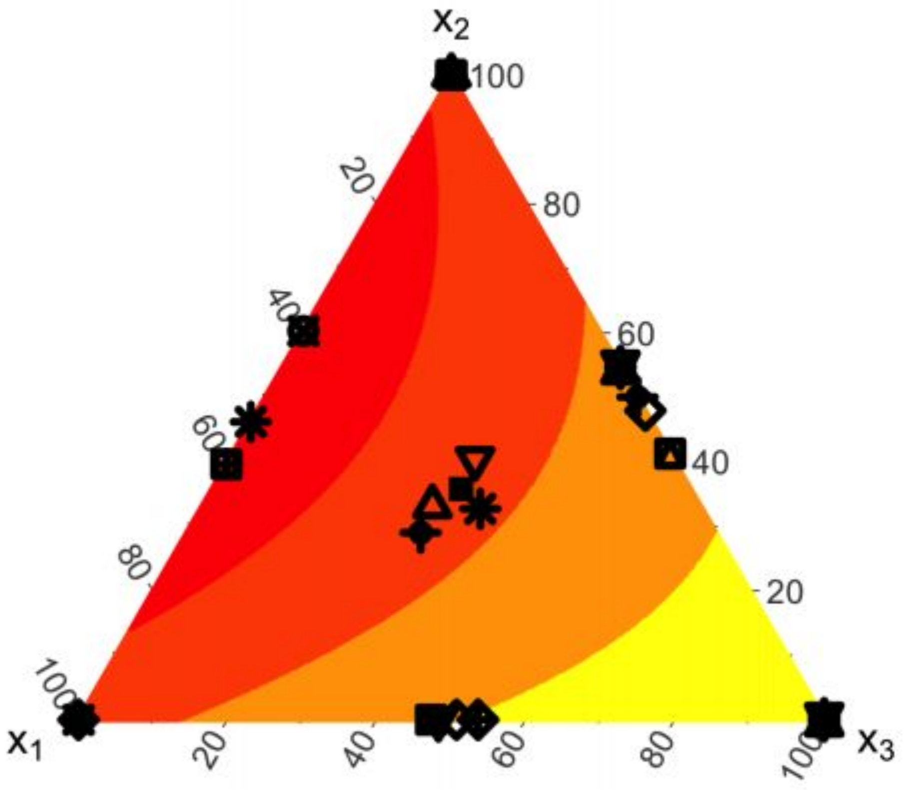
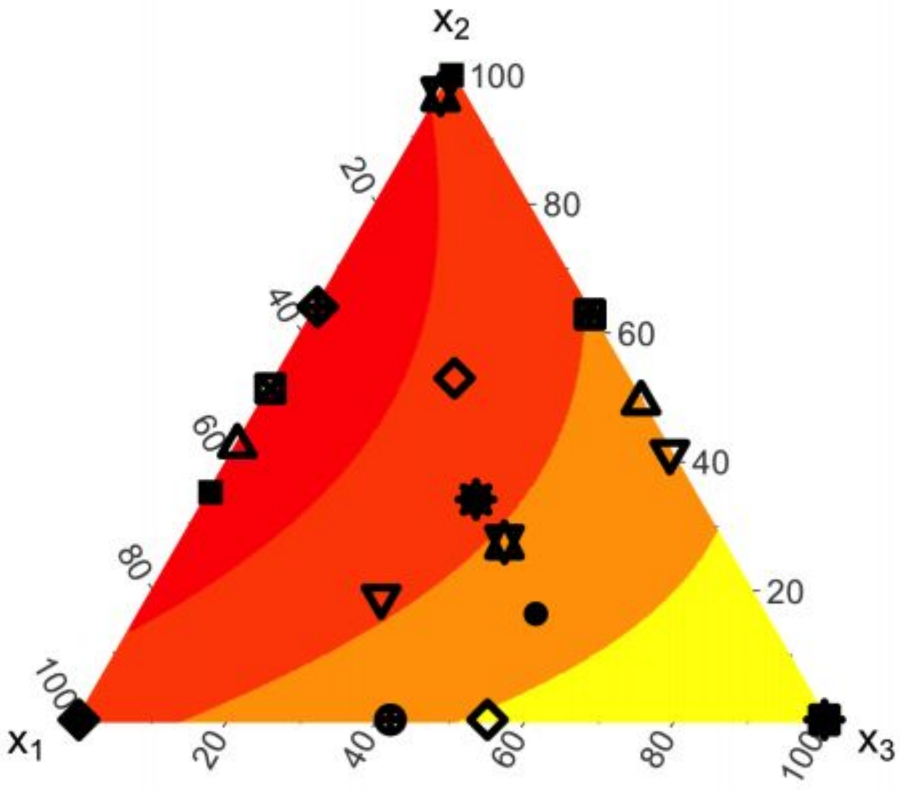


Bayesian D-optimal design



Bayesian I-optimal design

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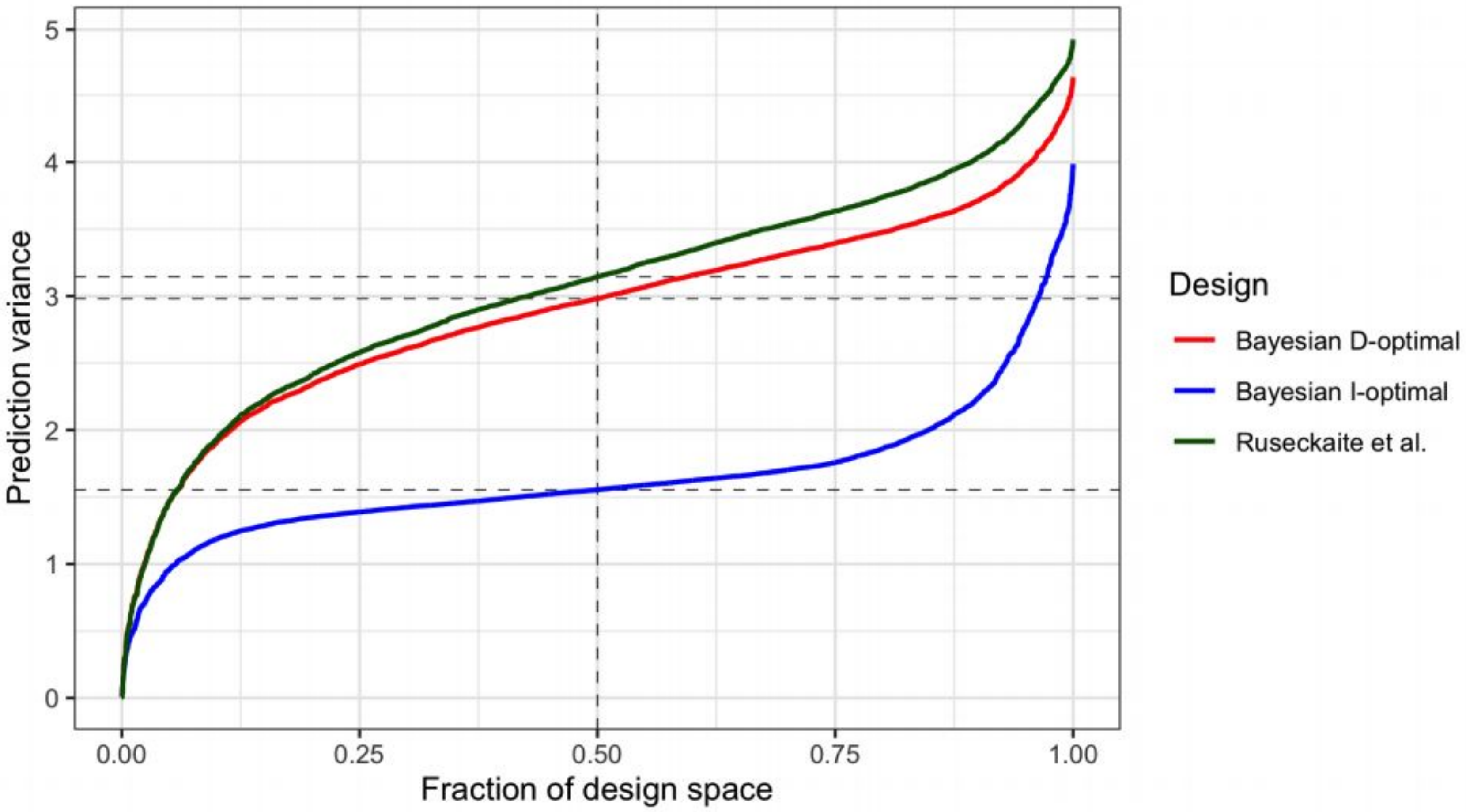

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- Models that take into account possible presence of consumer heterogeneity

More information

- Becerra, Mario, and Peter Goos. *Bayesian I-optimal designs for choice experiments with mixtures.* Chemometrics and Intelligent Laboratory Systems 217 (2021): 104395. DOI: 10.1016/j.chemolab.2021.104395

- Mario Becerra's website (with links to paper, R package, and code to reproduce the paper): mariobecerra.github.io/

Thank you