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Bayesian D- and I-optimal designs for choice experiments with mixtures using a multinomial logit model

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Outline

1. Choice modeling and choice experiments
2. Mixture experiments
3. Combining choice models and mixture models
4. Optimality criteria for choice experiments
5. Results
6. Conclusions and future work

Discrete choice experiments

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- Respondents are presented sets of alternatives (choice sets) and asked to choose
- Example: a customer responding whether they prefer to buy product A, B or C
- Models assume a latent utility function used to derive the probability of each respondent making each decision

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  - ingredients used to make a cocktail

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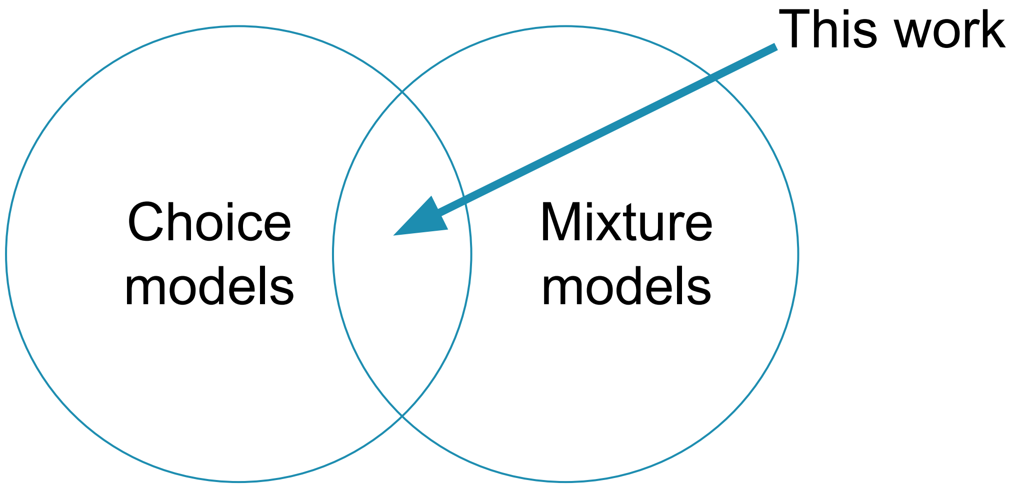
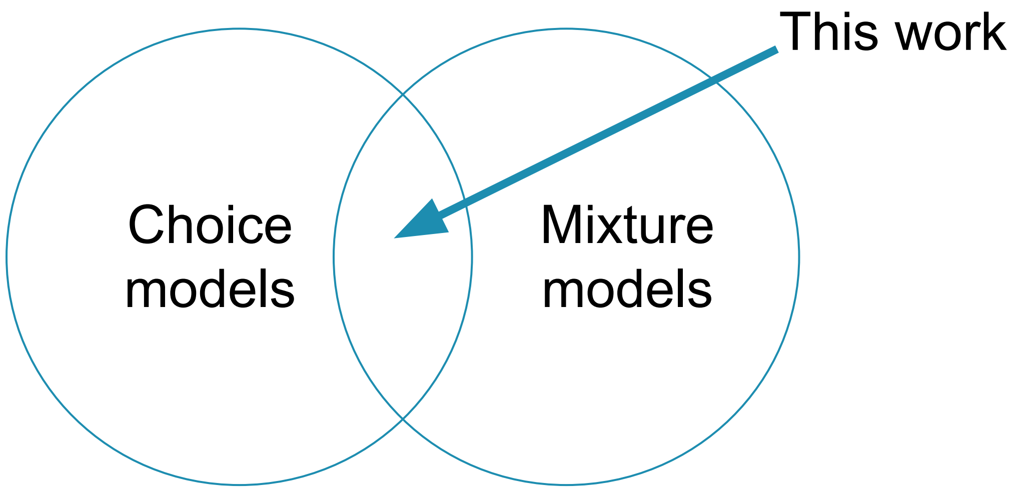
- The researchers' interest is generally in one or more characteristics of the mixture
- For us, the characteristic of interest is the **preference** of respondents
- Choice experiments are ideal to collect data for quantifying and modeling preferences for mixtures

Choice experiments with mixtures



Choice experiments with mixtures

- First example by Courcoux and Séménou (1997)



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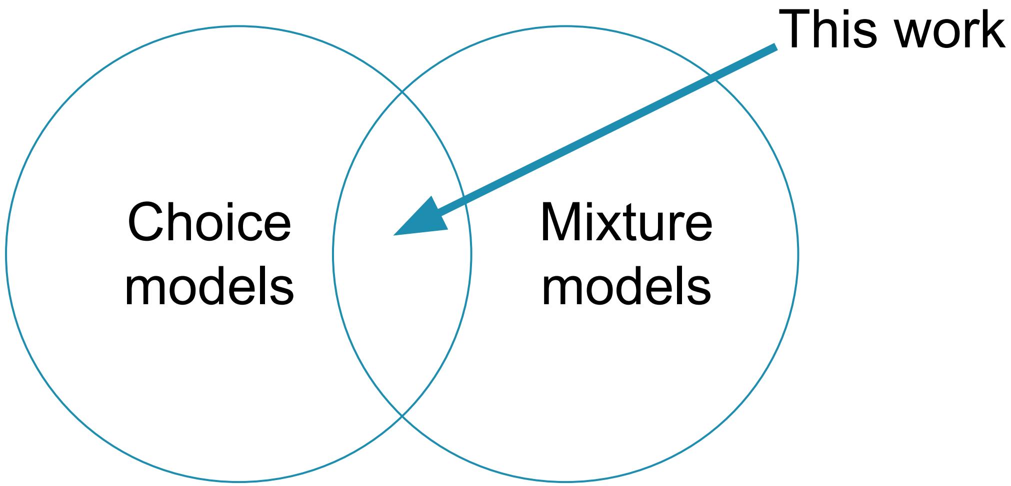

Choice experiments with mixtures

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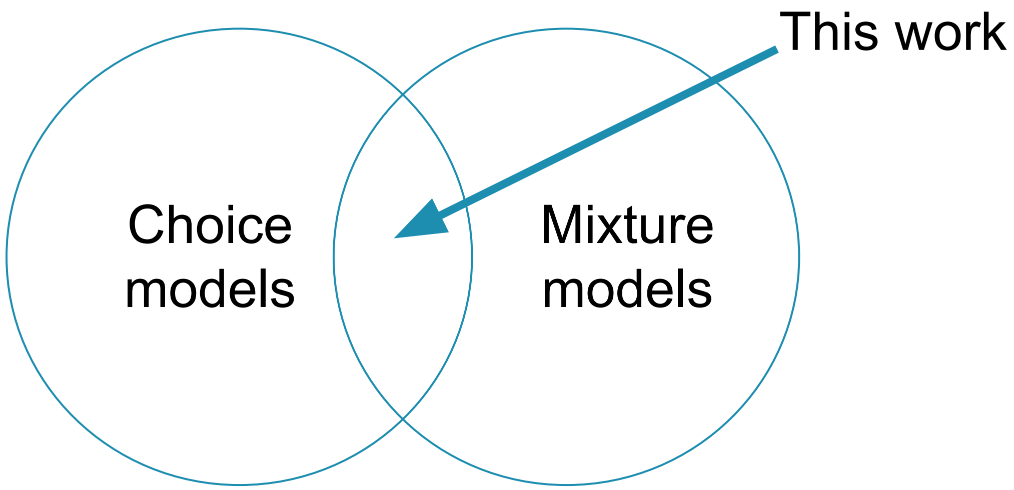
- Preferences for cocktails involving different proportions of mango juice, lime juice, and blackcurrant syrup
- Experimental data involved the responses of sixty people, each making eight pairwise comparisons of different cocktails



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- Optimal design of experiments is the branch of statistics that deals with the construction of efficient experimental designs

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- I-optimal designs are more suitable

Models for data from mixture experiments

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- Each mixture is described as a combination of *q* ingredient proportions, with the constraint that these proportions sum up to one
- Dedicated models are needed to avoid perfect collinearity
- Special-cubic Scheffé model:

$$Y = \sum_{i=1}^{q} \beta_i x_i + \sum_{i=1}^{q-1} \sum_{j=i+1}^{q} \beta_{ij} x_i x_j + \sum_{i=1}^{q-2} \sum_{j=i+1}^{q-1} \sum_{k=j+1}^{q} \beta_{ijk} x_i x_j x_k + \varepsilon$$

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- The probability that a respondent chooses alternative $j \in \{1, ..., J\}$ in choice set s is

$$p_{js} = \frac{\exp\left[\boldsymbol{f}^T(\boldsymbol{x}_{js})\boldsymbol{\beta}\right]}{\sum_{t=1}^{J} \exp\left[\boldsymbol{f}^T(\boldsymbol{x}_{ts})\boldsymbol{\beta}\right]}$$

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- Perceived utility modeled as

$$U_{js} = \boldsymbol{f}^T(\boldsymbol{x}_{js})\boldsymbol{\beta} = \sum_{i=1}^{q-1} \beta_i^* x_{ijs} + \sum_{i=1}^{q-1} \sum_{k=i+1}^{q} \beta_{ik} x_{ijs} x_{kjs} + \sum_{i=1}^{q-2} \sum_{k=i+1}^{q-1} \sum_{l=k+1}^{q} \beta_{ikl} x_{ijs} x_{kjs} x_{ljs} + \varepsilon_{js}$$

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- Bayesian D-optimality criterion

$$\mathcal{D}_B = \log\left(\int_{\mathbb{R}^r}\left[\det\left(\boldsymbol{I}^{-1}(\boldsymbol{X},\boldsymbol{\beta})\right)\right]^{\frac{1}{r}}\pi(\boldsymbol{\beta})d\boldsymbol{\beta}\right)$$

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$$\mathcal{I} = \mathrm{tr}\left[\boldsymbol{I}^{-1}(\boldsymbol{X}, \boldsymbol{\beta})\boldsymbol{W}_u\right]$$

$$\boldsymbol{W}_u = \int_{\chi} \boldsymbol{f}(\boldsymbol{x}_{js})\boldsymbol{f}^T(\boldsymbol{x}_{js})d\boldsymbol{x}_{js}$$

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- 60 consumers were asked to taste different pairs of seven fruit cocktails
- Ruseckaite et al. obtained a prior distribution for parameter vector $\boldsymbol{\beta}$ in a special-cubic Scheffé model
- We used the same prior distribution to compute Bayesian D- and I-optimal designs using a coordinate-exchange algorithm

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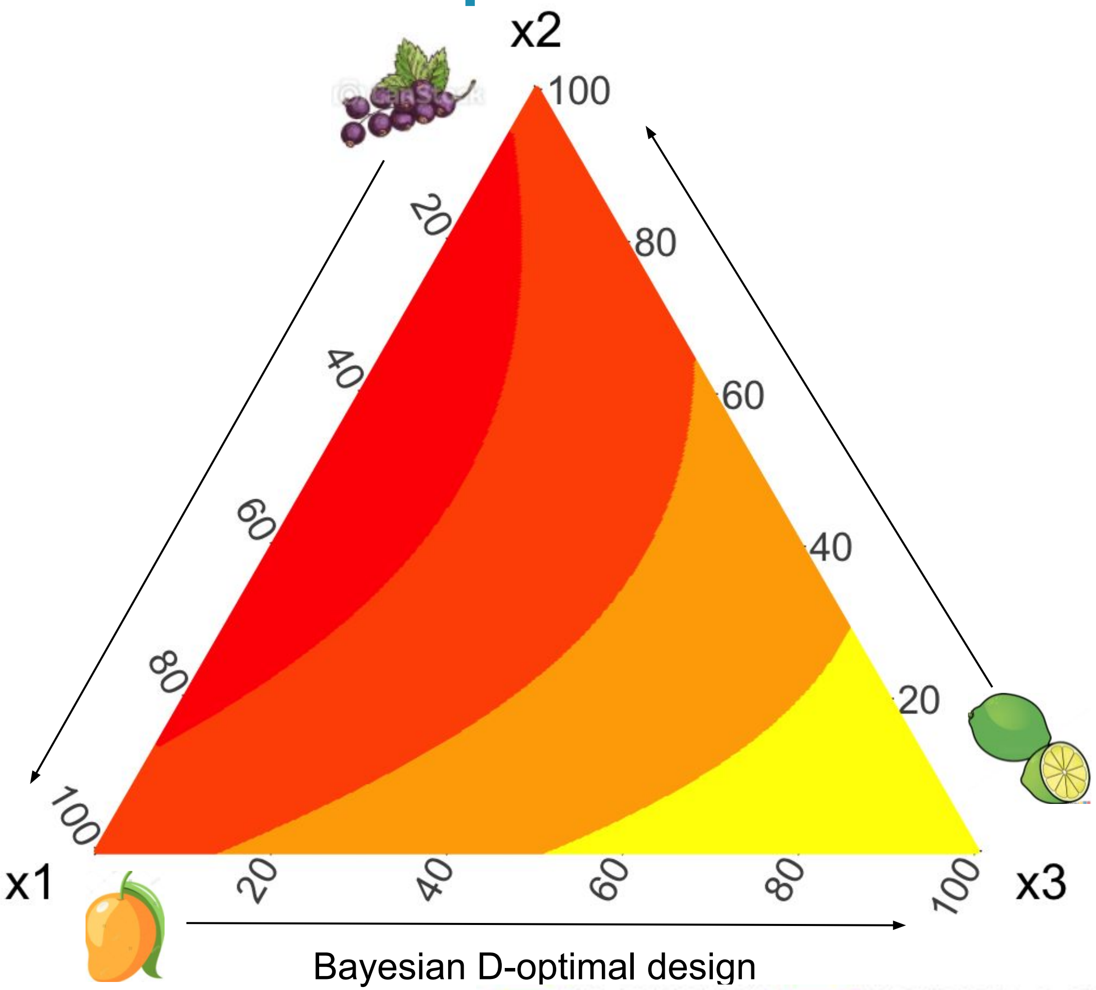
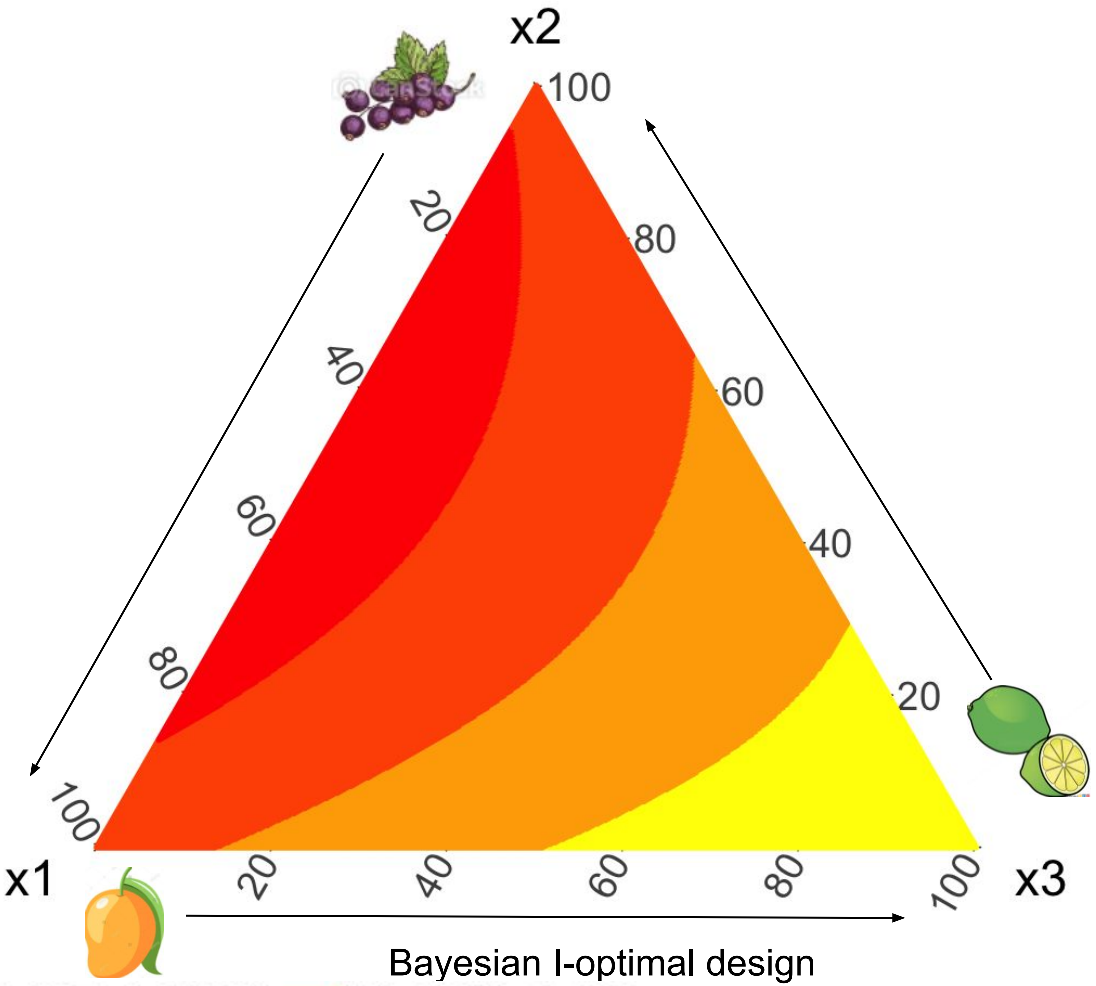


Bayesian D-optimal design



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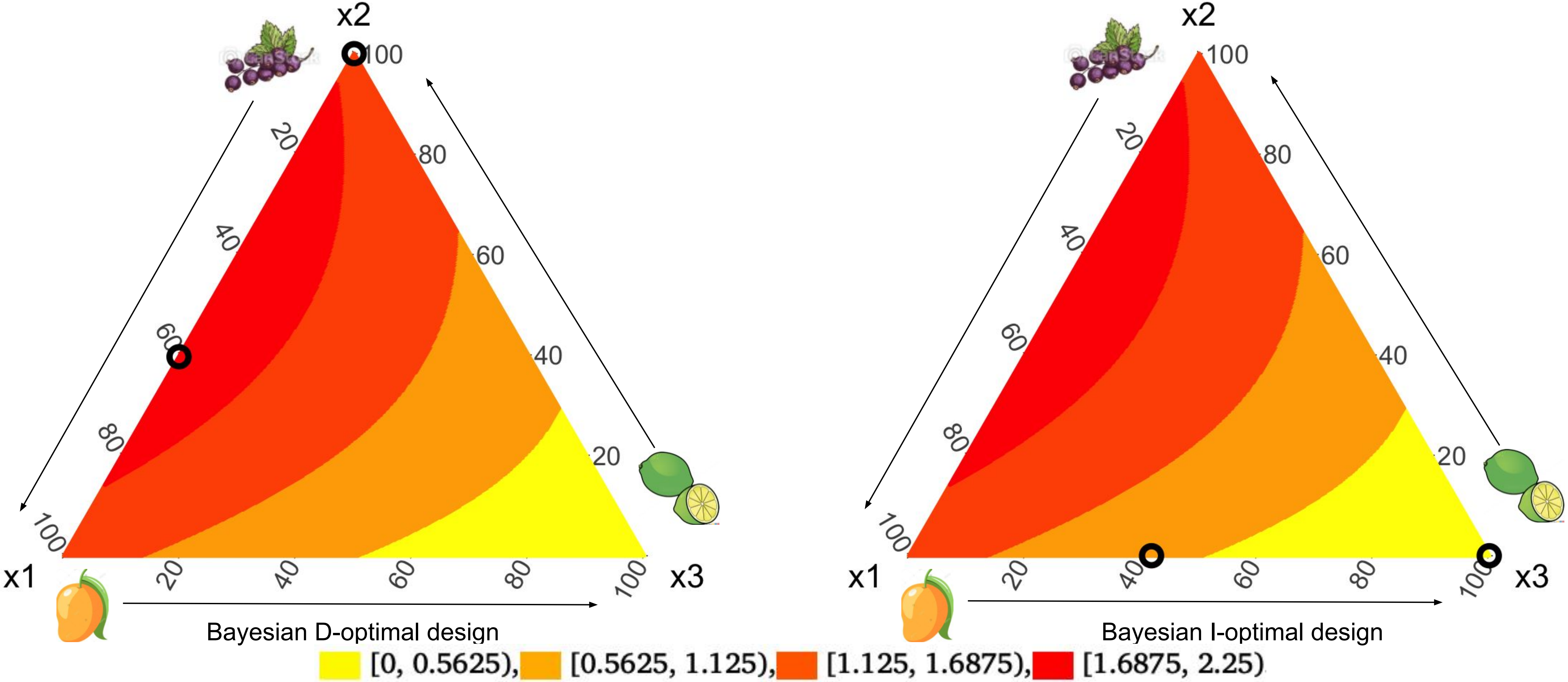

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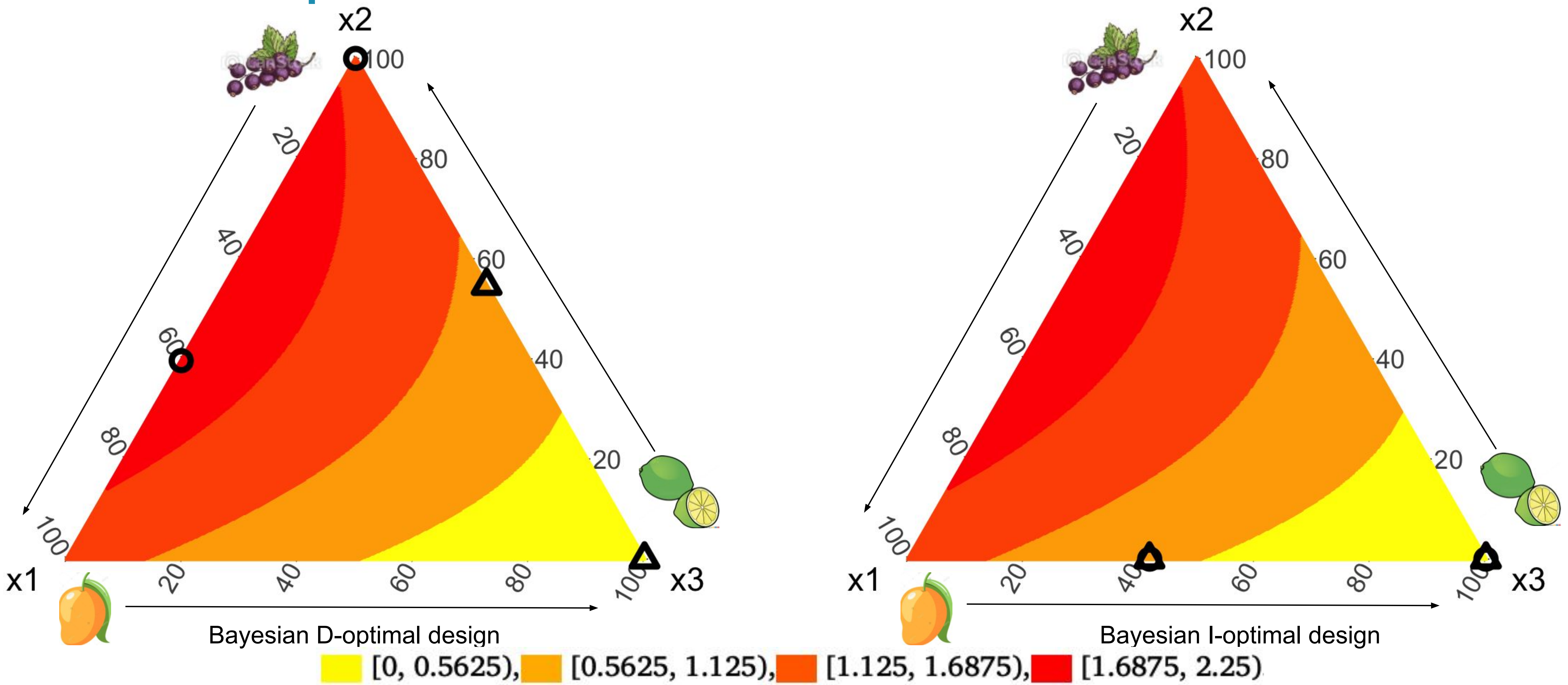

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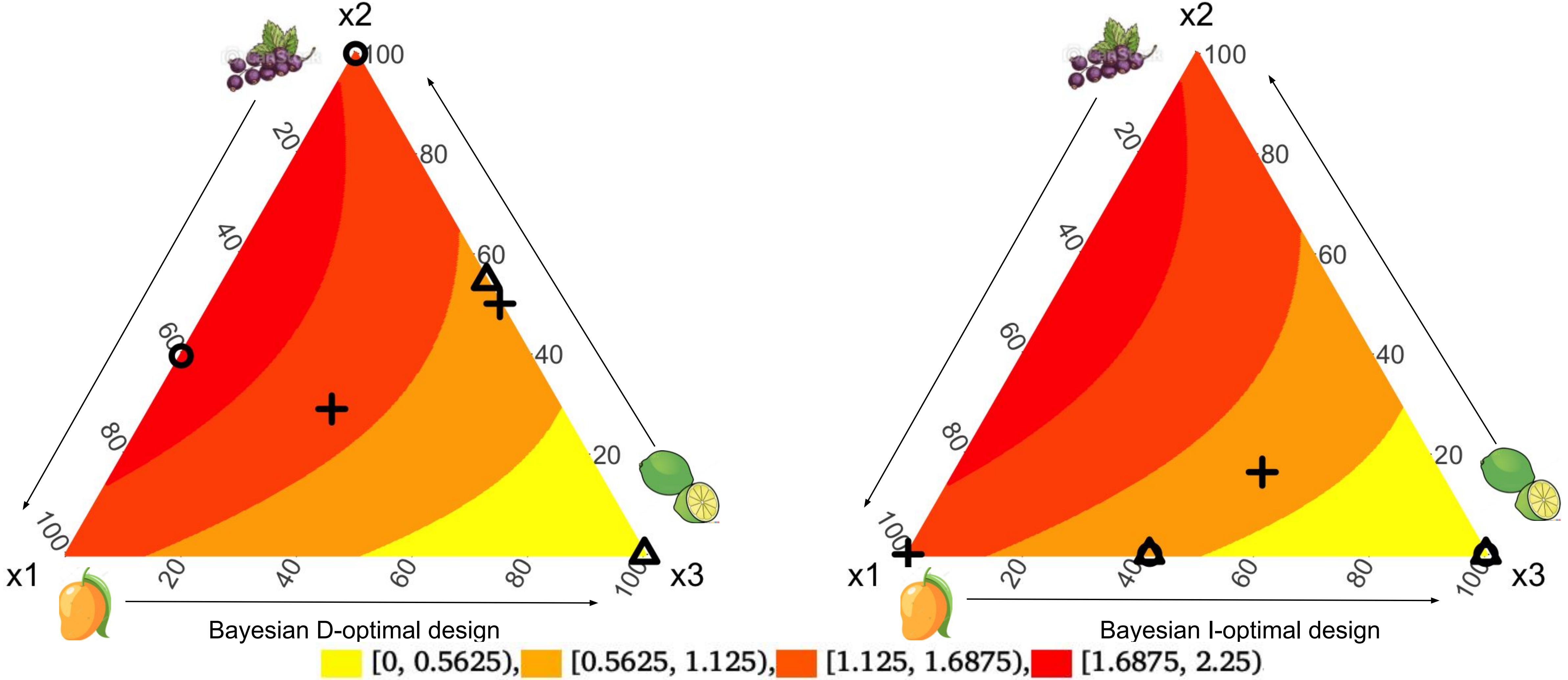

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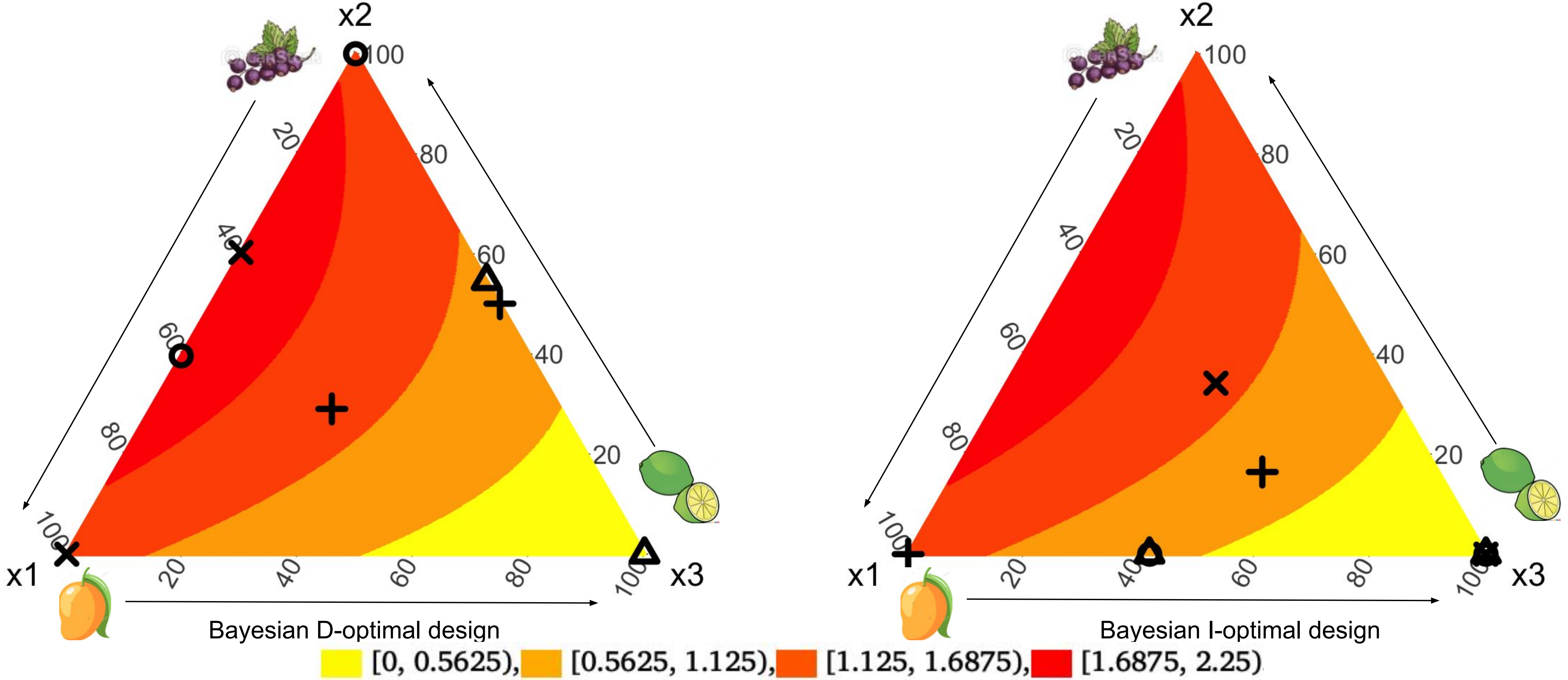

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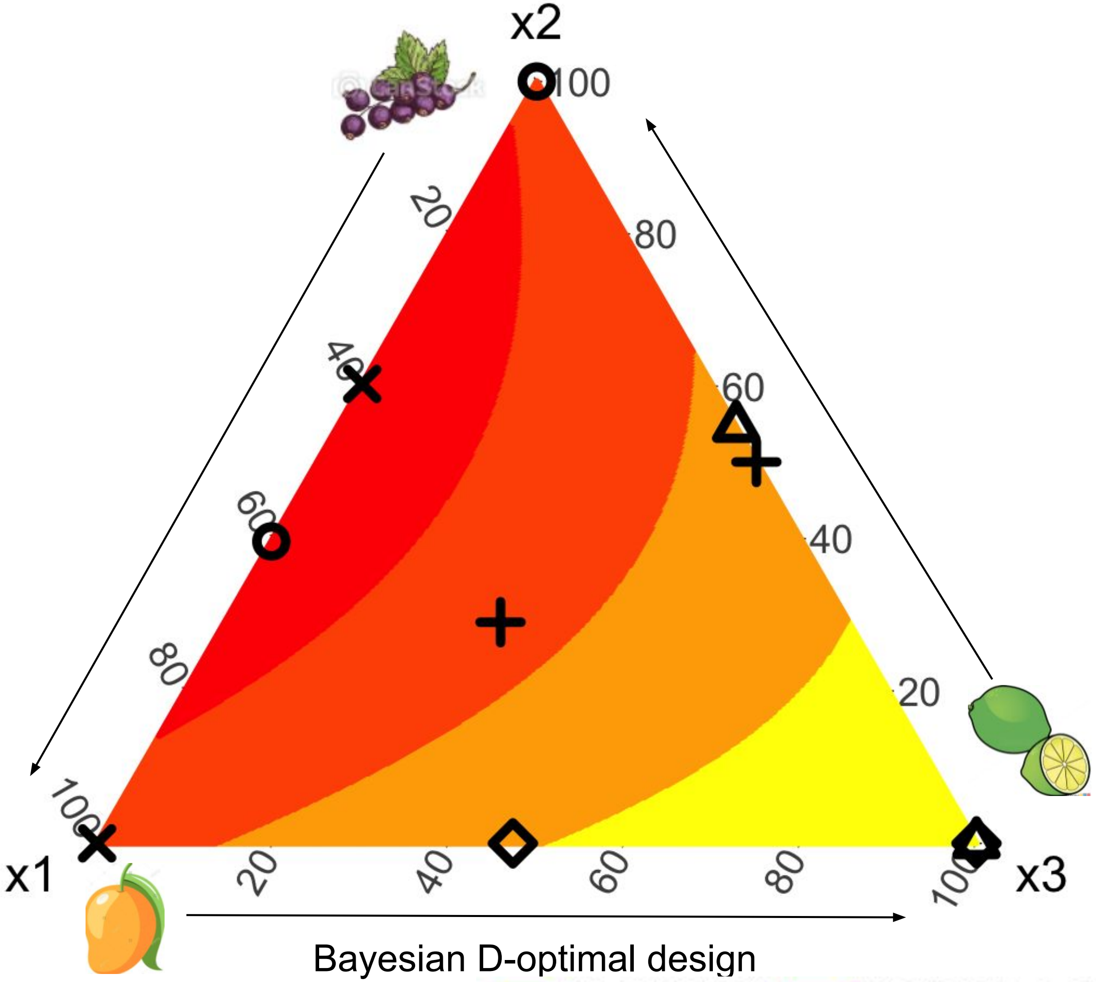

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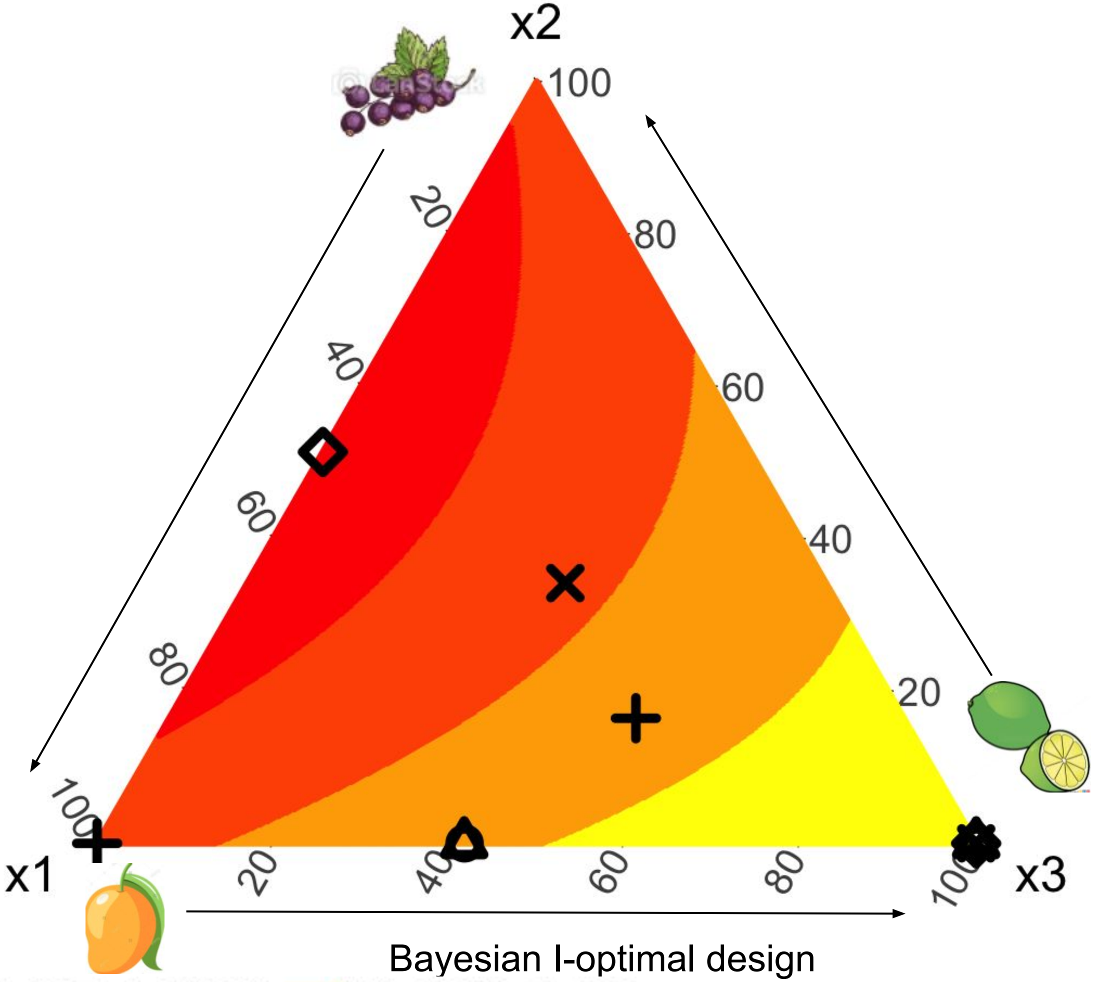

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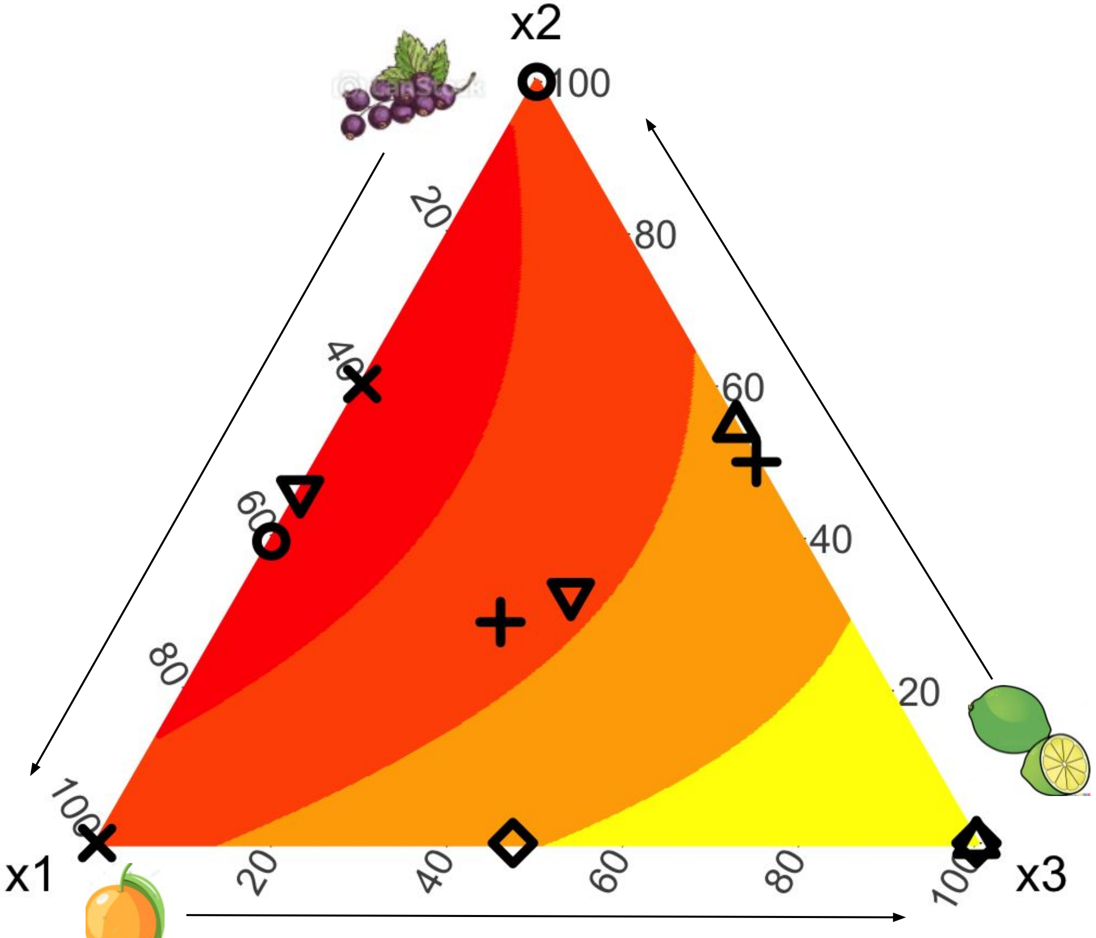

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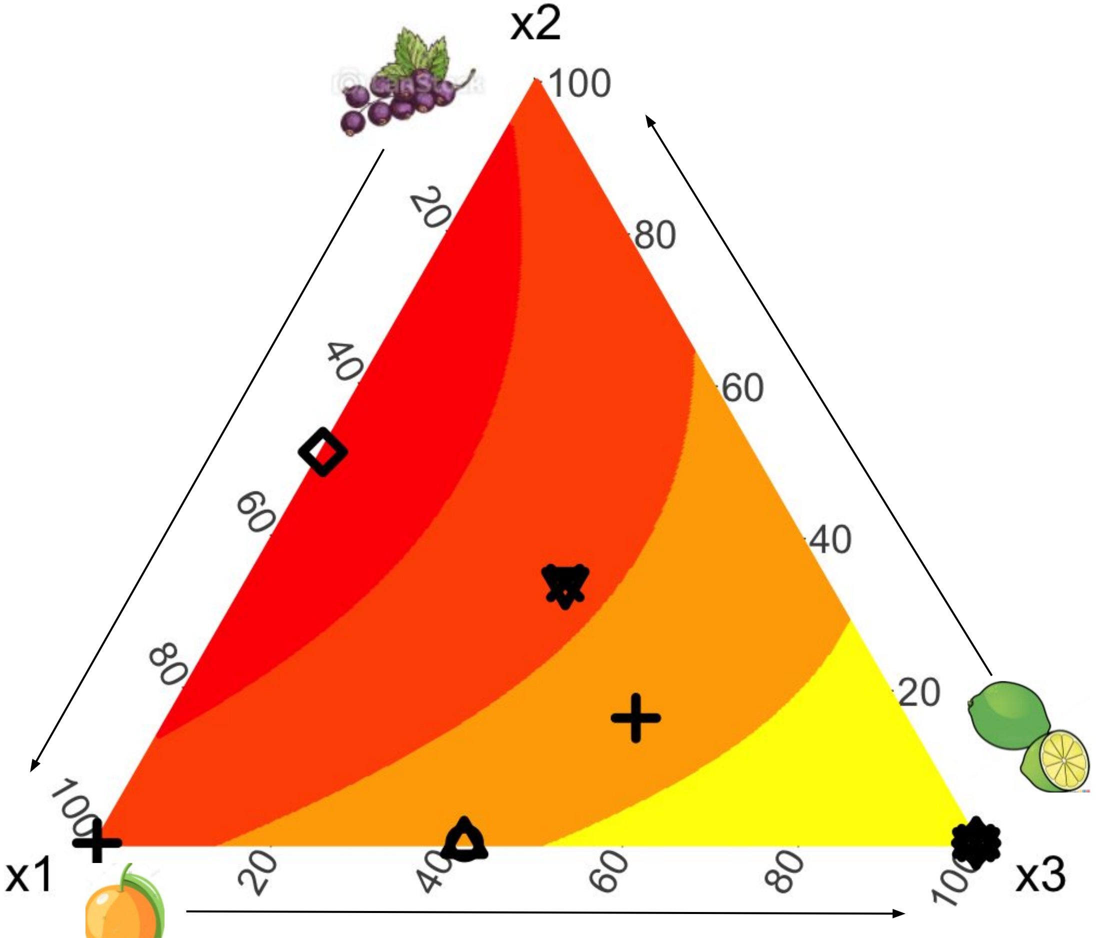

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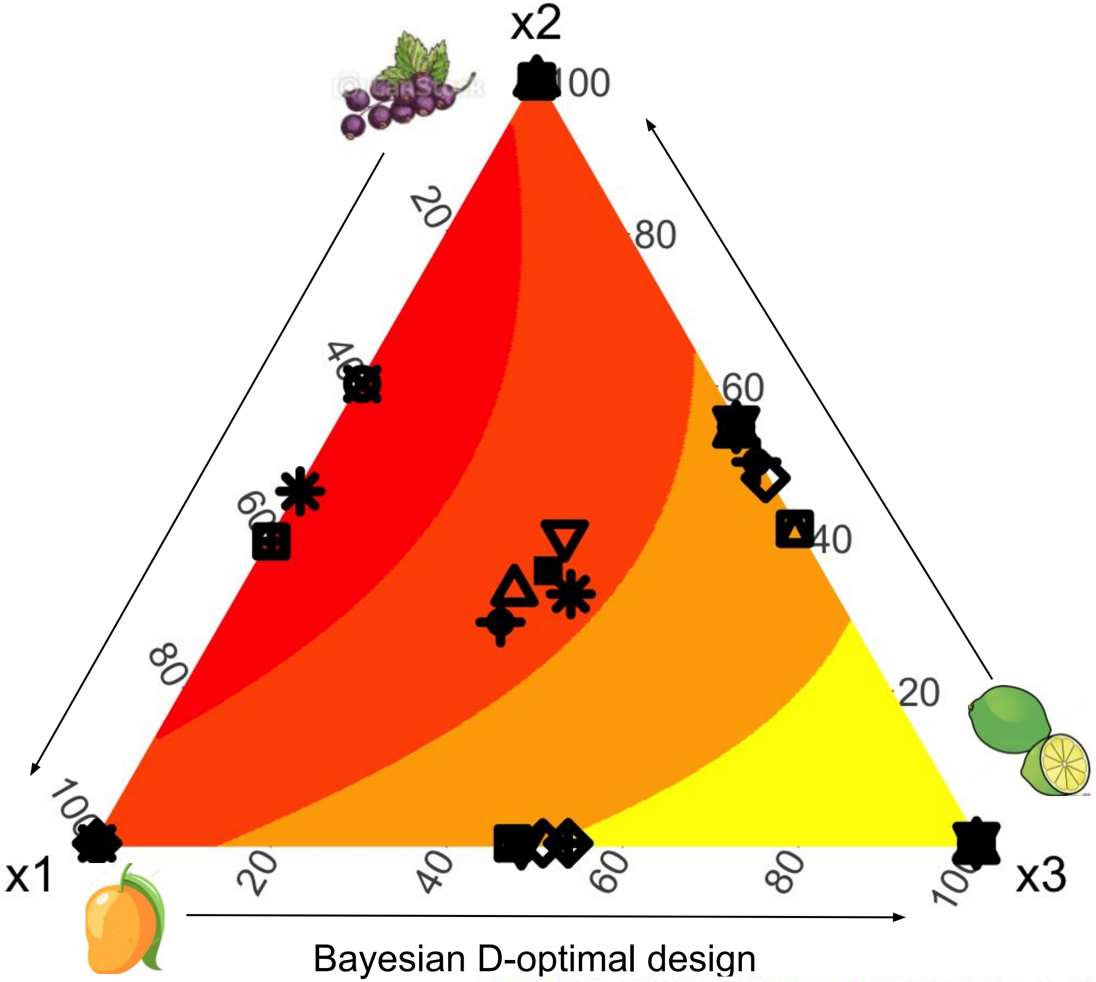

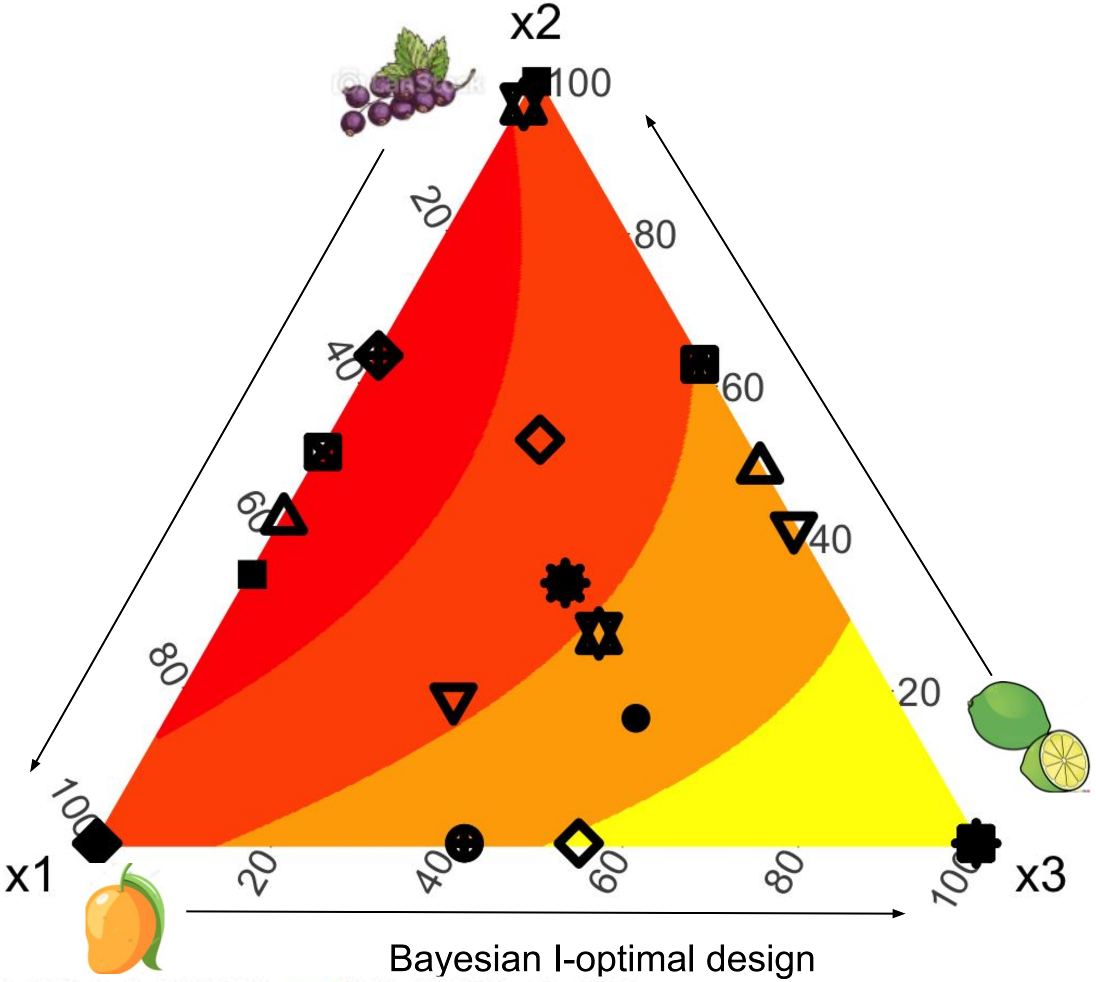

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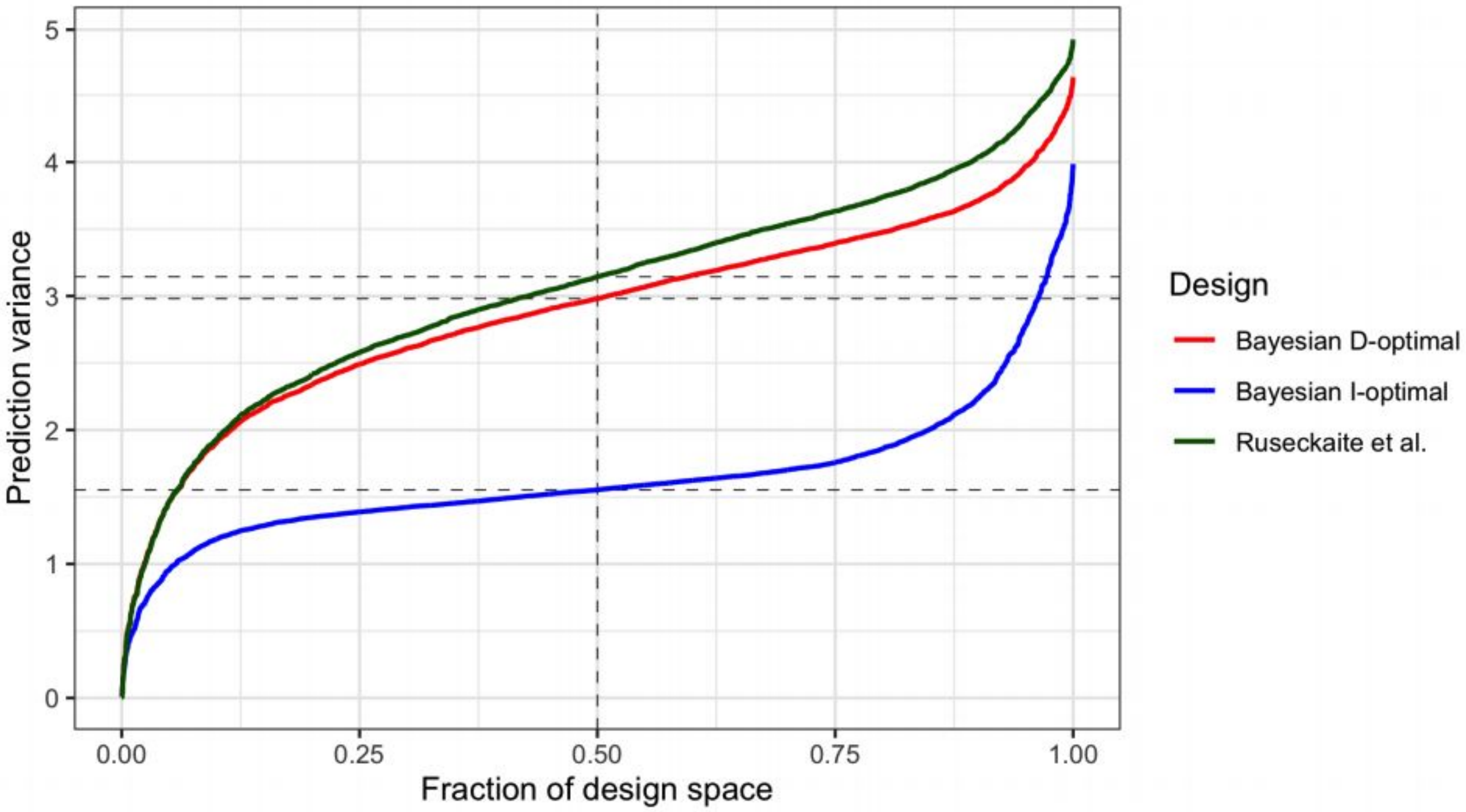

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- Add process variables
- Models that take into account possible presence of consumer heterogeneity
- Extend the work to other classes of models for data from mixture experiments

More information

- Becerra, Mario, and Peter Goos. *Bayesian I-optimal designs for choice experiments with mixtures.* Chemometrics and Intelligent Laboratory Systems 217 (2021): 104395. DOI: 10.1016/j.chemolab.2021.104395

- Mario Becerra’s website (with links to paper, R package, and code to reproduce the paper): mariobecerra.github.io/